

# Currencies, Competition, and Clans

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## Abstract

We present a theoretical and empirical analysis of stable coexistence among the world's anchor currencies (G3): the dollar, euro and yen. The theoretical model presented in this paper builds on a model of spatial competition and rests on a set of assumptions related to the behavior of central banks, the workings of exchange rate regimes, the geography of money, and international monetary arrangements. We show that stable coexistence in the sense of the pure-strategy equilibrium derived in our model is attainable in the case of two anchor currencies, but not in the case of three. The empirical evidence provides support for the assumptions and conclusions of the model.

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## 1. Introduction

Since the collapse of the Bretton Woods system the world of money has been dominated by three anchor currencies (G3): the U.S. dollar, the Japanese yen, and the euro (formerly the Deutsche mark).<sup>1</sup> With respect to their significance, several major proposals have suggested that the G3 countries enter into a joint target zone that would stabilize their exchange rates around estimated equilibrium levels within specific bands (see Volcker, 1997; McKinnon, 1997; and Williamson, 1998) or constitute a basis for a new international currency (Mundell, 2001, 2005). The reasons behind these proposals reflect worries related to global monetary stability (Salvatore, 2007) and global structural imbalances (Salvatore, 2006).

Cohen (1998) analyzed the power of anchor currencies by forging the concept of “authoritative domain”, which combines the extent to which an anchor currency facilitates transactions and the territoriality of its use. Authoritative domain thus combines the functional as well as spatial dimensions of an anchor currency into a single concept of use and authority. The size of an anchor currency’s authoritative domain grows when additional “satellite” currencies are linked to the anchor currency via an exchange rate regime and form an informal currency area. An anchor currency captures a satellite currency when the latter is tied to the former via some more or less strict exchange rate regime and through this informal currency area the size of an anchor currency’s authoritative domain grows.<sup>2</sup> The options for ties between anchor and satellite currencies range from a simple peg to managed float with an anchor currency as a

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<sup>1</sup> Given the steadily growing importance of the Deutsche mark among the European currencies after World War II, we take the liberty of extending the Deutsche mark to the euro since such a continuity greatly simplifies the empirical part of this paper. There is no other motivation beyond this.

<sup>2</sup> “Only a privileged few states with the most widely circulated currencies, such as the U.S. dollar, Europe’s new euro (succeeding Germany’s Deutsche mark), and the Japanese yen, can realistically aspire to a unilateralist leadership strategy”, writes Cohen (2004, p. xv). In terms of the yen, McKinnon (2000) asserts that central banks in North and Southeast Asia maintain unaltered soft dollar pegs. On the other hand, several studies find a declining influence of the dollar in the region (Kwan, 1996; Aggarwal and Mougue, 1996; Bowman, 2005). Kearney and Muckley (2005) provide a review of the previous research on the influence of the Japanese yen in Asian foreign exchange markets. Most recently, Kearney and Muckley (2007a) examine the evidence of an emerging yen block in North and Southeast Asia using up to 27 years of weekly data on nine bilateral yen exchange rates. Their results indicate a strong but declining influence of the U.S. dollar. Their findings are consistent with the documented rise in intra-regional trade integration in North and Southeast Asia, and with an emerging yen influence that falls short of a de facto yen block. In a further reassessment, Kearney and Muckley (2007b) document that the yen’s influence is rising amongst a subset of the Asian currencies since the early 1990s. Thus, besides the fact that Japan itself intervenes heavily in the dollar from time to time, and that the yen/dollar rate is not a pure float, the above literature supports the fact that the yen qualifies as an anchor currency.

reference currency.<sup>3</sup> *Clans* in the title of the paper refer to sets containing anchor currency and captured satellite currencies. A currency is not “captured” when a satellite currency follows a pure float.

Why would an anchor currency be interested in extending its authoritative domain in the first place? It is common knowledge that the G3 currencies are vehicle currencies whose function as an exchange medium extends far beyond domestic trade to wider use in international transactions. As a consequence, monetary transactions and their volumes represent a transmission mechanism that provides the central bank of an anchor currency with control over satellite currencies within an informal currency area. The authority is due to the fact that as an exchange rate arrangement becomes more tightly tied to the anchor currency, the central bank of a satellite currency becomes less independent in monetary policy conduct and the origin of its monetary base becomes more foreign. The trade-off for decreased independence is reduced exchange rate volatility and a potential for lower inflation. The reason is because the authority over satellite currencies is not a one-way process. A tie between satellite currencies and an anchor currency greatly reduces volatility within an informal currency area. Reduced volatility, in turn, may promote international trade by further reducing the costs of business activities for economies of both anchor and satellite currencies.<sup>4</sup> In addition, if satellite currency countries tie their currencies to an anchor currency with lower inflation, they are also likely to benefit from reduced inflation.<sup>5</sup> To summarize, central banks of anchor currency countries have practical reasons to be interested in capturing satellite currencies and increasing their authoritative domain, because this allows them to gain greater

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<sup>3</sup> A comprehensive discussion of a wide range of currency regime changes (actual and potential) around the world is given by Cohen (2004). In terms of exchange rate arrangements, there has been growing support for abandoning regimes between hard pegs and free floats because of high capital mobility. Such a bipolar view is frequently referred to as a “corner solution” or the “hollowing out of intermediate regimes.” It was initially discussed by Eichengreen (1994); relevant arguments were further provided by Obstfeld and Rogoff (1995), Goldstein (1999), and Eichengreen and Fisher (2001). Proponents of the bipolar view argue that pegs or floats are the only regimes compatible with the current degree of market integration and that countries with high capital mobility should discard intermediate regimes in favor of extreme ones.

<sup>4</sup> The advantages of a common currency, such as lower transaction costs in trade and less uncertainty about relative prices were recognized already by Mundell (1961). McKenzie (1999) reviews the impact of exchange rate volatility on international trade. Using the conditional variance approach Kroner and Lastrapes (1993) find a negative effect of volatility on trade for the U.S. and the UK. More recent literature contains pronounced evidence on the negative impact of exchange rate volatility on international trade. Based on a gravity model approach López-Córdova and Meissner (2003) and Dell’Aricia (1999) find evidence that exchange rate volatility has a negative impact on international trade.

<sup>5</sup> The importance of low inflation is theoretically grounded as well as empirically documented (see Giavazzi and Giovannini 1989, among others).

control over satellite currencies, to reduce volatility and hence promote international trade, and to increase price stability. Satellite countries, in turn, import some of these benefits through pegs to the anchor.

The purpose of this paper is to analyze the stable coexistence, in the sense of a pure-strategy equilibrium, among the three anchor currencies that govern current monetary affairs. First, we introduce a two-stage game to model the dynamics of how anchor currencies compete for satellite currencies. Second, we operationalize the theoretical concept of authoritative domain developed by Cohen (1998). Third, we provide empirical evidence to illustrate whether a tripolar currency world would, in fact, provide a workable framework to achieve the desired stable coexistence among the anchor currencies.

To accomplish the above goals we formally analyze the relationship among the G3 currencies with the aid of a stylized model that is a modification of the spatial competition model of Hotelling (1929). Our model is based on the simple and widely recognized premise that a central bank's objective is price stability. Price stability depends on two factors, the anchor currency's central bank policy and the size of its authoritative domain. This implies a trade-off for the anchor currency's central bank when choosing its policy. Changing the policy may directly increase price stability, but at the same time it can reduce the authoritative domain and thus indirectly negatively affect price stability. The interest rate, as the main factor in a latent one-dimensional policy instrument, is used to conduct central bank policy.<sup>6</sup> The model is set up as a two-stage game in which the central banks of the anchor currencies compete for shares of the currency holdings of satellite countries that represent a proxy for the size of the authoritative domain.<sup>7</sup> The preferences of the satellite countries for the policies of the anchor currency's central bank to which they are linked are assumed to be distributed along a line as in the Eaton and Lipsey (1975) model. We show that with some changes in assumptions, the results of the standard spatial competition model continue to hold. Specifically, we find for a large range of parameters a pure-strategy equilibrium in the case of two anchor currencies, but not in the case of three.<sup>8</sup> This suggests that exchange-

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<sup>6</sup> Such a latent variable may reflect the constraints of a central bank, which we do not consider for the simplicity of our model.

<sup>7</sup> The monetary policy of a satellite country is assumed to have a negligible impact.

<sup>8</sup> As our model is highly stylized and focuses on long-term behavior, it does not address other aspects of an

rate stability is achievable with two anchor currencies but generally not with three anchor currencies. We note that the purpose of the model is not intended to provide a precise explanation of the observed patterns of exchange rate development, but rather to present an illustration of the possible consequences if the number of anchor currencies is reduced from three to two. The empirical part of the paper provides evidence that our assumptions are realistic and that the observed phenomena are consistent with our model's predictions.

The paper is organized as follows. In Section 2 we present and analyze the formal theoretical model. Section 3 describes the data and provides empirical extensions. A brief conclusion follows in Section 4.

## **2. Model and Equilibrium Analysis**

Cohen (1998) introduced the notion of a currency's "authoritative domain" by combining the functional dimension (transactions) as well as the spatial dimension (territoriality) of money into "a single amalgam of use and authority." The authoritative domain of a foreign anchor currency expands with the number and, more specifically, volume of satellite currencies that are tied via various exchange rate regimes to the anchor currency. In effect, when the central bank of a satellite currency pegs its currency to an anchor currency, it cedes control over the value of the satellite currency to the anchor foreign currency. Under these circumstances, satellite currencies do not have reason to disappear, though their authoritative domain is greatly eroded. Consequently, transactions and their volumes represent an economic transmission mechanism that gives the central banks of anchor currencies control over satellite currencies because "the currency of a country that has a large share of international output, trade and finance has a natural advantage" (Jeffrey Frankel as quoted in Cohen, 1998, p. 97).

Given this reality of the international geography of money, we build a model of spatial competition among the central banks of anchor currencies in a two-stage game setup. There are  $n$  anchor currencies, each attached to one large country (or to a group of countries that form a monetary union). In addition, there is a continuum of satellite countries, each with its own currency. A satellite country is defined by its monetary policy having only a negligible influence on world markets. The policy space of the

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anchor currency central bank's policies.

central banks of the anchor currencies is one-dimensional. This one-dimensional policy space is indeed the result of a set of policy choices, but for simplicity we collapse it into one single variable: the interest rate, which is the dominating policy instrument as well as the most significant loading factor of our formal generalization.<sup>9</sup> Within this policy space, there is a range that fulfills the basic goals of the central bank. We normalize this feasible policy space to  $[0,1]$ .<sup>10</sup> Following the above approach we aim to take a broad perspective on international currencies and downsize the fine details highlighted in the literature on international currencies. Still, international trade, exchange rates, markets and prices are present in our analysis in the background of our model set-up and its motivations.

The objective of an anchor central bank in our model is price stability. An independent central bank prefers domestic policy autonomy to exchange rate management, as it has no socio-political incentives to produce competitive, stable exchange rates. Its goals are predominantly to achieve low domestic inflation.<sup>11</sup> Indeed, in reality, usually price stability and, hence, some type of inflation management is one of the explicit goals of a central bank. Implicitly, central banks may be concerned about economic growth or the trade deficit, since these are related to the bank's foreign exchange reserves. Thus, these goals also serve to increase price stability, albeit indirectly.<sup>12</sup>

Using standard theory, the origin of a monetary base can be inferred from a country's choice of exchange rate regime. If a country favors a floating exchange regime, then the

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<sup>9</sup> In the real world the importance of the interest rate is accentuated by the behavior of large institutional investors on the financial markets through the phenomenon known as "carry trade", which is a form of investment strategy among leveraged investors and real money managers. The strategy exploits the forward bias by investing in high-yielding currencies. In a "carry trade", an investor borrows in a low interest rate currency and then takes a long position in a higher interest rate currency betting that the exchange rate will not change so as to offset the interest rate differential.

<sup>10</sup> Replacing this interval with an open interval does not change the results. Further, given that our policy variable is assumed to primarily reflect the interest rate, our one-dimensional policy space is better characterized as a line and not, for example, as a circle, as a circular policy space would imply that very high and very low interest rates correspond to the same policy. We address generalizations to a multi-dimensional policy space below.

<sup>11</sup> Baines (2001) documents three trends in the political economy of exchange rate policy in advanced industrialized countries: an unprecedented rise in capital mobility, a favor of floating exchange rates (at least officially) and a need for sound monetary policy, and high levels of central bank independence. These trends result in monetary policy directed at maintaining domestic price stability above all other concerns. In our model we employ price stability as the central bank's objective.

<sup>12</sup> In any event, in our stylized model we restrict our attention to the single and arguably dominant motive of price stability. In reality, specific goals vary across central banks but we prefer to keep the model tractable over maximizing its realism. We might not lose much in terms of realism due to our simplification.

monetary authority has, by definition, full control over its monetary policy, no exchange rate policy, and a monetary base whose origin is entirely domestic. On the other hand, if a country prefers to peg its domestic currency to a foreign one, then the central bank de facto resigns from an independent monetary policy, conducts an explicit exchange rate policy, and has a monetary base of purely foreign origin. Any exchange rate regime between the two extremes means a different extent of independence in both its monetary and exchange rate policies as well as a mixed origin of the monetary base. Hence, by knowing the (true) adopted exchange regime we may identify the amounts of domestic money (of satellite currencies) linked to a particular anchor currency via the exchange rate regime, and express this amount in terms of the anchor currency. The sum of the above amounts may be understood as a proxy for the extent of the anchor currency's authoritative domain or, conversely, for the dependency of satellite currencies.

We define the dependencies of satellite currencies on anchor currencies in the context of arguments given by Reinhardt and Rogoff (2004). Based on their categorization of de facto (true) exchange regimes, we are able to trace the shares of foreign currency holdings. In this way we can proxy for the anchor currency's authoritative domain more precisely than relying on official exchange regime categorization, which often does not reflect reality. Formally, let  $C$  be the amount of domestic currency expressed in terms of foreign anchor currencies to which a domestic currency is linked via a particular exchange rate regime, and  $c_i$  be the part of  $C$  expressed in anchor currency  $i$  that corresponds to the weight of  $i$  in the currency basket. Clearly,  $\sum_{i=1}^n c_i = C$ . This convenient notation covers all the possible cases outlined above:

- 1) when  $n=0$ , then  $C=0$  and the satellite currency is floating;
- 2) when  $n=1$ , then the satellite currency is pegged to an anchor currency; and
- 3) when  $n>1$ , then the satellite currency is under a currency basket peg regime.<sup>13</sup>

Hence, the central banks of anchor currencies attract, through their policy choice, satellite currencies that tie with anchor currencies via exchange rate regimes. Satellite countries have a preference for the location in the policy space of the anchor currency to which they link their (satellite) currencies. The most preferred locations of satellite currencies are distributed with respect to a density  $f$  on  $[0,1]$ . Satellite countries'

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<sup>13</sup> More details on the construction of monetary aggregates are given in section 3 in conjunction with our empirical assessment.

preferences differ because their economic conditions differ, i.e. while some are exclusively interested in price stability, others might prefer a somewhat less restrictive policy in order to stimulate growth.

Our basic assumption is that price stability for a large country  $i$  (with an anchor currency) depends on two factors: the policy (interest rate) of the central bank  $x_i$ , and the aggregate share  $s_i$  of domestic currency, expressed in the anchor foreign currency  $i$ , that is held by satellite countries whose domestic currency is linked via a specific exchange rate regime to anchor currencies. More precisely, the objective function of a central bank is  $G_i(x_i, s_i)$ , where  $G_i$  is a proxy for price stability, and is increasing in  $s_i$  but decreasing in the absolute difference between its actual policy  $x_i$  and its preferred policy  $p_i$ .<sup>14</sup> Therefore, when choosing its policy, an anchor currency's central bank has to consider not only the direct effect on price stability, but also the indirect effect via the change in the share of satellite currencies linked to it.<sup>15</sup>

We analyze this interaction between the central banks of anchor currencies and those of satellite countries as a two-stage game. In this game, the central banks of  $n$  anchor currencies first decide simultaneously on their policy, i.e. on their location in the policy space  $x_1, \dots, x_n$ , and then the satellite countries choose their foreign currency holdings.

Our model resembles the spatial competition model by Eaton and Lipsey (1975), but differs in three respects. First and most importantly, we introduce the preferences of anchor currencies' central banks over their location in the policy space. Second, as will be seen below, satellite countries do not exclusively choose the anchor currency closest to their own preferred policy, but rather a mix of respective closest currencies on both sides such that the weighted average policy of these currencies corresponds to the preferred policy. Finally, central banks can choose identical policies, in which case the linked

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<sup>14</sup> The objective function  $G_i$  is increasing in  $s_i$  since the connection of satellite currencies to an anchor currency via exchange rate regimes decreases the volatility of bilateral exchange rates in the informal currency area, and hence fosters international trade and reduces the costs of business activities. Since the economic conditions in anchor currency countries differ, not considering the effects of their policy choices on satellite countries' pegs, they have different preferences with respect to their policy.

<sup>15</sup> Devereux, Shi, and Xu (2007) detail a model of monetary policy under a U.S. dollar standard and describe how to conduct monetary policy once an anchor currency captures satellite currencies. Their work potentially can motivate our own, as well as lend support to our model that, besides other things, describes how to get to the point when satellite currencies are linked to anchor currencies via foreign exchange standards.

countries choose baskets with equal shares in these currencies.<sup>16</sup> We show below that the two central results of the basic spatial competition model, namely the existence of a pure-strategy equilibrium in the case of two anchor currencies and the non-existence of a pure-strategy equilibrium in the case of three anchor currencies, continue to hold if the costs of deviating from the preferred policy  $p_i$  are not too high and if the preferred policies of central banks of anchor currencies are relatively homogeneous compared to the distribution of the preferred policies of satellite countries.

Consider first the behavior of the satellite countries in the second stage. We assume that they aim to peg to a basket such that the weighted average of the interest rates of the anchor currencies in the basket is as close as possible to their preferred policy  $l$ . The underlying reason for this assumption that a satellite country would be interested in having an interest rate close to the mix of anchor currencies in the basket is rooted in interest rate parity theory. Equal interest rates do not constitute a ground for either exchange rate depreciation or appreciation and similar interest rates imply only low pressure on exchange rate adjustments over time. The central bank of a satellite currency country would therefore prefer to peg to an anchor currency (basket) that most closely reflects its preferred policy because this policy would otherwise not be sustainable or frequent exchange rate adjustments would have to be made. For the same reason, a satellite country prefers to include in the basket anchor currencies whose policy choice is closer to their own preferred policy. If a satellite currency country has a choice between two different baskets that have the same weighted average policy, it chooses the basket that minimizes the maximal absolute difference between  $l$  and the policies of the anchors included in the basket.

Given the satellite countries' preferences over the baskets they could peg to, their optimal choice will be a mix of the closest anchor currencies. More precisely, as above let  $C$  be the amount of domestic currency expressed in terms of foreign anchor currencies to which a domestic currency is linked via a particular exchange rate regime, and  $c_i$  be the part of  $C$  expressed in anchor currency  $i$  that corresponds to the weight of  $i$  in the currency basket ( $\sum_{i=1}^n c_i = C$ ). Without loss of generality, assume  $x_1 \leq \dots \leq x_n$ . If  $l \leq x_1$  then the country will choose a currency basket consisting only of currency 1,  $c_1 = C$ ; in

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<sup>16</sup> Such behavior can be observed during periods of post-war development and was a prominent feature of emerging economies during the last two decades of the 20th century.

such a case the currency basket reduces to a simple peg. If  $l \geq x_n$  then the country will choose  $c_n = C$ . If  $x_i \leq l \leq x_{i+1}$  then the country will choose a mix of currencies  $i$  and  $i+1$ ,  $c_i = C \frac{x_{i+1}-l}{x_{i+1}-x_i}$ ,  $c_{i+1} = C \frac{l-x_i}{x_{i+1}-x_i}$ . Note that  $(c_i x_i + c_{i+1} x_{i+1})/C = l$  and that  $c_i = C$  if  $l = x_i$ . If  $x_{i-1} = x_i$  then  $c_{i-1} = c_i = \frac{C}{2} \frac{x_{i+1}-l}{x_{i+1}-x_i}$  and similarly if  $x_{i+1} = x_{i+2}$  then  $c_{i+1} = c_{i+2} = \frac{C}{2} \frac{l-x_i}{x_{i+1}-x_i}$  (and correspondingly if more than 2  $x$  are identical).<sup>17</sup>

Below, we will consider subgame-perfect equilibria, where in each subgame following a choice of policies by the anchor currency countries, satellite countries will choose their best reply, i.e. choose their baskets as above. Thus when we discuss different equilibria below, we will only describe the behavior of the anchor currency countries explicitly.

Let us now turn to the first stage of the game, the anchors' choice of locations in the policy space. Assume for simplicity that  $C$  is identical for all satellite countries and normalize  $C = 1$ . Without loss of generality, this can be achieved by replacing the density of satellite countries  $f$  by the density of foreign currency-expressed holdings  $f^*$  with  $f^*(l) = \frac{f(l)C(l)}{\int_0^1 f(z)C(z)dz}$  for all  $l \in [0,1]$ , where  $C(l)$  denotes the average currency holding of the countries whose preferred location is  $l$ . Denote by  $s_i$  the share of currency  $i$  of the total foreign currency-expressed holdings by satellite countries.

Each anchor currency's central bank has a preferred policy  $p_i$ . As noted above, the aim of an anchor currency's central bank is to maximize  $G_i(x_i, s_i)$ , where  $G_i$  is assumed to be linear increasing in  $s_i$  but the costs of deviating from the preferred policy  $p_i$  (henceforth "location costs") is convex in the absolute difference. More precisely, let  $G_i(x_i, s_i) = s_i - L(x_i - p_i)$  with  $L(-y) = L(y)$ ,  $L'(0) = 0$  and  $L''(y) > 0$ .<sup>18</sup> Assume furthermore for simplicity that the preferred policies of satellite countries are distributed according to a uniform distribution on  $[0,1]$  and that location costs are quadratic,  $L(y) = a(y)^2$  with  $a > 0$ .<sup>19</sup> Since the leading economies are more alike each other than all economies are, the preferred policies of the central banks of anchor currency countries

<sup>17</sup> Modern monetary history documents that the number of currencies in a basket usually ranges from 2 to 5. The basket of currencies within the former EMS is an exception due to the institutional setup.

<sup>18</sup> Note that the cost function is the same for all anchor currencies, only  $p_i$  differs.

<sup>19</sup> We will elaborate below on generalizations.

are assumed to be relatively similar compared to the distribution of the preferences of satellite currencies' central banks. Furthermore, it appears that small deviations from the preferred policies have a relatively small impact on relocation compared to  $s_i$ ; hence  $a$  is assumed to be small enough such that concerns for location costs do not dominate concerns for the share of currency holdings  $s_i$ .

**Proposition 1:** Let there be two anchor currencies and let their preferred policies be  $p_1 < p_2$ . Then:

- (a) There is an equilibrium  $x_1 = x_2 = \frac{1}{2}$  if  $p_1 \geq \frac{1}{2} - \frac{1}{4a}$  and  $p_2 \leq \frac{1}{2} + \frac{1}{4a}$ .
- (b) If  $p_2 - p_1 > \frac{1}{2a}$  and  $|1 - p_1 - p_2| \leq a(p_2 - p_1)^2 - \frac{1}{4a}$ , then  $(x_1, x_2)$  with  $x_1 = p_1 + \frac{1}{4a}$  and  $x_2 = p_2 - \frac{1}{4a}$  forms an equilibrium.
- (c) If the other cases do not hold, there is no equilibrium in pure strategies.

The proof extends the logic of the basic Hotelling game to the case with location costs. It involves nothing but checking systematically that under the given conditions no central bank of the anchor currencies has an incentive to deviate, while in all other constellations of policies at least one of them does have an incentive to deviate. The proof follows the standard steps and is omitted for the sake of space (available upon request).

While the proof is somewhat tedious, the results are intuitive. Part (a) says that if the preferred policies are close enough to the median of the distribution of small countries, such that the marginal location costs are smaller than  $\frac{1}{2}$  at the median (note that if  $x_1 \neq x_2$ , the absolute value of the derivative of  $s_i$  with respect to  $x_i$  is  $\frac{1}{2}$ ), then the classical result that both central banks choose the location at the median survives. Obviously, this is the only equilibrium where both central banks choose the same policy since otherwise a marginal deviation would lead to an increase in  $s_i$  at essentially zero location costs. The range for  $p_1$  and  $p_2$  such that  $x_1 = x_2 = \frac{1}{2}$  in an equilibrium is decreasing in  $a$ . Part (b) says that if the preferred policies are sufficiently far apart, both central banks will choose policies such that marginal location costs are equal to the marginal gains in  $s_i$ , i.e.  $\frac{1}{2}$ . Note that this implies that the chosen policies are closer together than the preferred policies. The condition  $|1 - p_1 - p_2| \leq a(p_2 - p_1)^2 - \frac{1}{4a}$  ensures that neither of the two banks has an incentive to “pass” the other bank. In order for such an equilibrium to exist, the preferences of the central banks of anchor currency countries

have to be very different, but the range where such an equilibrium exists is increasing in  $a$ .

The above logic also applies to more general distributions of the preferred policies of satellite countries and to more general convex location costs. In particular, if location costs are sufficiently small and the preferred policies of the two central banks of anchor currencies are relatively close to the median of  $f$ , then location costs are dominated by concerns for the share of satellite currencies and the minimal differentiation result holds, i.e. there is an equilibrium where both choose policies equal to the median. If the preferred policies are rather different and location costs are high, then location costs dominate concerns for shares in satellite currencies and there is an equilibrium where they choose different policies.

Note that since the conditions for the equilibria of type (a) and (b) are mutually exclusive, the pure-strategy equilibrium (if it exists) is unique. Hence, in a finitely repeated game the equilibrium play will be repeated and the situation is stable in the sense that the policies of the two anchor currencies are stable over time and satellite countries do not change their exchange rate regimes. In the case of an equilibrium of type (a) the policies will change if, due to external shocks, the preferred policies of the satellite countries shift. They will, however, change in a parallel fashion provided that the shift is not too radical, because the equilibrium policies will stay at the median as long as the condition in (a) remains fulfilled. The policies will, however, not change if the preferred policies of the central banks of anchor currencies shift as long as condition (a) holds. But they will shift if the equilibrium is of type (b). Furthermore, if the preferred policies of the central banks of anchor currencies move closer together over time, we can move from an equilibrium of type (b) to one of type (a) (if the preferred policies are relatively symmetric to the median of  $f$ ) or to the non-existence of a pure-strategy equilibrium (if they are highly asymmetric). In an infinitely repeated game, Folk-theorem arguments imply that we get additional equilibria. Repeated play of the stage-game equilibrium is, however, also one equilibrium of the infinitely repeated game such that if the conditions in (a) or (b) hold, a stable pattern would be one equilibrium (and the only one that does not require any punishment threats to be stabilized).

**Proposition 2:** Let  $n = 3$  and  $p_1 < p_2 < p_3$ . Then:

$$(a) \text{ If } 1) p_2 - p_1 \geq \frac{1}{4a}, 2) p_3 - p_2 \geq \frac{1}{4a},$$

$$3) \max\{p_1 + \frac{p_2}{2} - \frac{p_3}{2} + \frac{7}{16a}, -\frac{p_1}{2} - p_2 + \frac{p_3}{2} - \frac{3}{16a}\} \leq a(p_2 - p_1)^2, \text{ and}$$

$$4) \max\{1 + \frac{p_1}{2} - \frac{p_2}{2} - p_3 + \frac{7}{16a}, -1 - \frac{p_1}{2} + p_2 + \frac{p_3}{2} - \frac{3}{16a}\} \leq a(p_3 - p_2)^2,$$

then  $(p_1 + \frac{1}{4a}, p_2, p_3 - \frac{1}{4a})$  is an equilibrium.

(b) Let  $x_3 = p_3 - \frac{1}{4a}$  and  $x_1 = \frac{x_3}{3}$ . If  $p_2 \leq x_1 \leq p_1 + \frac{1}{4a}$  and

$$5) 1 \leq \frac{8}{9}p_3 + \frac{8}{9}ap_3^2 - \frac{5}{18a} + \frac{p_2}{3} - \frac{4}{3}ap_2p_3, \text{ then } (x_1, x_1, x_3) \text{ is an equilibrium.}$$

(c) Let  $x_1 = p_1 + \frac{1}{4a}$  and  $x_3 = \frac{x_1+2}{3}$ . If  $p_3 - \frac{1}{4a} \leq x_3 \leq p_2$  and

$$6) 2 \geq 8p_1 + 4ap_1 - 8ap_1^2 + 4a + \frac{5}{2a} + 3p_2 - 12ap_2 + 12ap_2p_1,$$

then  $(x_1, x_3, x_3)$  is an equilibrium.

This is just a mirror image of case (b).

(d) If the other cases do not hold, there is no pure-strategy equilibrium.

Like for proposition 1, the proof is somewhat tedious but it involves only checking systematically that given the above conditions, no central bank has an incentive to deviate, while in all other constellations of policies, at least one bank has an incentive to deviate. The proof again follows the standard steps and is omitted for the sake of space (available upon request). While more technical assumptions are needed here than in Proposition 1 to ensure that in the given equilibrium no bank would like to deviate to a position just marginally beyond the position of one of the other banks, the main results are again intuitive. In case (a) of Proposition 2, conditions (1) and (2) ensure that if banks 1 and 3 move to the positions where marginal location costs are equal to the marginal gains in the share (i.e.  $\frac{1}{2}$ ), bank 1 is still to the left of bank 2 and bank 3 to the right of bank 2. Conditions (3) and (4) ensure that bank 2 does not want to deviate to  $x_1 - \varepsilon$  or  $x_3 + \varepsilon$ . Since bank 2 cannot change its share by moving between banks 1 and 3, the only such constellation that is an equilibrium is that bank 2 chooses its preferred policy. Such an equilibrium exists only if the anchor currency banks' preferences are highly heterogeneous but the range of parameters such that an equilibrium of type (a) exists is increasing in  $a$ . By contrast, cases (b) and (c) involve situations where two banks whose preferred policies are relatively close, will choose the same policy—resembling the formation of a monetary (policy) union—while the third bank whose preferred policy is very different, will choose a policy such that its marginal location costs are equal to the marginal gain in the share (i.e.  $\frac{1}{2}$ ).

Hence the existence of a pure-strategy equilibrium requires that at least one central bank has preferences very different from the other central banks (or that location costs are very high). Note in particular that there is no equilibrium where all three banks choose the same location. In this case, they would all receive  $s_i = 1/3$ , but a marginal deviation would allow a bank to capture at least  $s_i = 1/2$  at a negligible increase in location costs.

The qualitative results of Proposition 2 extend to more general convex cost functions and more general distributions of the preferences of satellite countries.<sup>20</sup> In particular, if the anchor currency central banks' preferences are relatively homogeneous compared to the preferences of satellite countries and location costs are not excessively high, there is no equilibrium in pure strategies. The logic is the same as in the standard spatial competition model: banks 1 and 3 would like to choose locations close to  $x_2$ . In that case  $s_2$  would be small, but bank 2 could increase  $s_2$  at only a small increase in location costs by deviating to  $x_1 - \varepsilon$  or  $x_3 + \varepsilon$ .

In a (finitely) repeated game the non-existence of a pure-strategy equilibrium means that the actual choices of central banks in period  $t$  do not form an equilibrium. Therefore, at least one central bank would like to change its policy. Hence, the configuration of locations of central banks of anchor currencies will change from period  $t$  to  $t + 1$ , even without external shocks and, moreover, not in a parallel fashion.

In other words, if there is no pure-strategy equilibrium there is only a mixed-strategy equilibrium and naturally the mixed strategies will (in general) yield different realizations and hence different locations of anchor currencies in each period. Thus we would expect fluctuating policies of the anchor currencies to follow a random pattern. As a consequence, the currency baskets of at least some of the satellite countries will also change from period to period.

The difference between the cases  $n = 2$  and  $n = 3$  can be summarized as follows. If the preferences of the central banks of anchor currencies are highly heterogeneous or location costs are very high, then for both  $n = 2$  and  $n = 3$  a pure-strategy equilibrium exists where central banks choose different policies. But if, as we assume, the preferences of central banks of anchor currencies are relatively similar compared to the distribution of

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<sup>20</sup> In the latter case,  $x_2 = p_2$  would in general not hold any more in a type (a) equilibrium, because density  $f$  is not constant and hence  $s_2$  is not the same for all  $x_2$  with  $x_1 < x_2 < x_3$ .

satellite countries' preferences and location costs are not very high, then the result of the model without location costs survives, namely that for  $n = 2$  there is an equilibrium where both central banks choose a policy at the median of the distribution of satellite countries' preferences, and if  $n = 3$ , then there is no equilibrium in pure strategies. In the latter case, the implementation of mixed-strategy equilibrium policies would follow a random pattern.<sup>21</sup> The crucial implication of the absence of a stable equilibrium among the policies of anchor currency countries is that exchange rates between these countries are unstable as well.

Although general results for  $n > 3$  can be derived by a similar extension of Eaton and Lipsey (1975), we do not address this here, because the empirically relevant cases are  $n = 2$  and  $n = 3$ . Eaton and Lipsey (1975) also show that the results are quite different for a higher dimensional choice space. This fact would most likely carry over to our model if we extended it to a multi-dimensional policy space. However, even if we considered a higher-dimensional policy space, our model would always be a substantial simplification. Hence, our results can only be an illustration of the possible impact of changes in the number of anchor currencies. Therefore, we prefer to adhere to the comparatively simple one-dimensional version of the model.

Our results also give insight into the notion of a currency's authoritative domain as explicated by Cohen (1998). We extend his arguments by showing that the existence of an equilibrium strongly depends on the number of competing anchor currencies.

### 3. Quantitative Evidence and Statistical Inference

The model designed in the previous section cannot be directly estimated by conventional econometric techniques. To illustrate its link to real economy phenomena we provide some quantitative evidence as well as statistical inference below.

We collected data on the exchange rates of domestic currencies with respect to the U.S. dollar, the Deutsche mark/ECU/euro,<sup>22</sup> and the Japanese yen (the anchor

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<sup>21</sup> Our results indicating a stable coexistence with two anchor currencies are also broadly in line with numerous theoretical models of foreign exchange trading that use a two-currency framework (see Matsuyama, Kiyotaki, and Matsui, 1993 and Zhou, 1997 among others). Moreover, the non-existence of an equilibrium in pure strategies in the case of three currencies corresponds to the results of Rey (2001), where the three-country model of the world economy has three partial and three total equilibria, where each currency can be the vehicle.

<sup>22</sup> ECU denotes European currency unit, a composite currency established with the inception of the European Monetary System.

currencies). Figure 1 shows the dynamics of the anchor currencies' exchange rates. These are defined as the percentage deviations from a standard value of 100 for March 1973, after the collapse of the Bretton Woods System. Furthermore, we assembled data on monetary aggregates (in terms of M2), short-term interest rates (discount rates of central banks), type of exchange rate regimes, inflation and aggregate output for 30 OECD countries plus Russia.<sup>23</sup> Because of their economic capacity and the derived amount of monetary aggregate used, we consider the OECD countries as a proxy for the world.<sup>24</sup> Short-term interest rates are defined as three-month money market rates, or rates on similar financial instruments. The span of our yearly data is 1964 to 2006, with the exception of emerging economies where meaningful data are available only from the mid-1980s. All data were assembled from OECD Economic Outlook statistics, IMF International Financial Statistics and, for particular missing data, from the central banks and finance ministries of the respective countries.<sup>25</sup>

We construct the sums of monetary aggregates (M2) of satellite currencies that are linked to anchor currencies based on the description of de facto (true) exchange regimes provided in Reinhardt and Rogoff (2004). In accord with our model in Section 2, we define  $C$  as the amount of domestic (satellite) currency expressed in terms of foreign anchor currencies to which a domestic currency is linked via the particular exchange rate regime;  $c_i$  is the part of  $C$  expressed in anchor currency  $i$  that corresponds to the weight of  $i$  in the currency basket ( $\sum_{i=1}^n c_i = C$ ).<sup>26</sup> This convenient notation covers all possible cases that are of interest: when  $n=0$ , then  $C=0$  and the satellite currency is floating; when  $n=1$ , then the satellite currency is pegged to an anchor currency; and when  $n>1$ , then the satellite currency is under a currency basket peg regime. If a country favors, for instance, a currency basket peg, then the weights of currencies in a basket are used to determine

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<sup>23</sup> The countries in our data sample are: Australia, Austria, Belgium, Canada, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Spain, Sweden, Switzerland, Turkey, the United Kingdom, the United States, and Russia.

<sup>24</sup> Because of this, we do not incorporate into our sample a number of satellite countries.

<sup>25</sup> Due to data inconsistencies we do not cover the 1950s in our analysis. This exclusion does not constitute a deficiency since the Bretton Woods System was firmly in place at that time and none of the repositioning implied by our model could take place.

<sup>26</sup> Technically,  $C$  should also include the foreign exchange reserves of the central banks of satellite currencies held in anchor currencies. However, since those foreign exchange reserves consist of currencies already issued by the central banks of anchor currencies, we cannot consider them. Aside from this, the structure of foreign exchange reserves held usually reflects the weights of the anchor currencies within the exchange rate regime.

the importance of anchor currencies with respect to satellite currency holdings. Since currencies in a basket usually represent those most frequently used in the conduct of international trade or international monetary operations of a specific country, such an approach is fully justified.

In particular, we specify the amounts of satellite currencies linked to a particular anchor currency via an exchange rate regime, and express these amounts in terms of the specific anchor currency. The sum of the above amounts along with the sum of the anchor currency itself form a reasonable proxy for the extent of an anchor currency's authoritative domain, despite the fact that "the data simply do not exist to accurately report all cross-border use of currencies, let alone more subtle relationships" as Cohen (1998, p. 24) accurately notes.<sup>27</sup> We trust that our construction is a realistic way to operationalize the concept of authoritative domain.

Figure 2 shows the dynamics of the monetary aggregates specified in the preceding paragraph during 1964–2006. The dynamics of monetary aggregates associated with the three currencies reflects the second stage of the game in our model. The total amount of monetary aggregate (M2) is divided into three groups according currencies' link to the U.S. dollar, to the Deutsche mark/ECU/euro, or to the Japanese yen. In accordance with historical developments we see a massive shift away from the U.S. dollar after the collapse of the Bretton Woods System, and a proportionally pronounced gain in Europe. While the share of currencies linked to the U.S. dollar stabilized in the late 1980s, the European currency has been steadily solidifying its share. The Japanese yen has a significant share of money linked to it, hovering below or around 30 per cent of the total after 1973.<sup>28</sup> In the context of our model the U.S. dollar clearly dominates from the 1960s to 1971–1973. This is when the number of anchor currencies is just one;  $n = 1$ . The period 1971–1979 represents a transition after the Bretton Woods System collapsed. We see a departure from state  $n = 1$  towards  $n > 1$ . During this period there exist no obvious candidates that would firmly establish a situation of two anchor currencies in which  $n =$

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<sup>27</sup> We acknowledge that there are other dimensions to an international currency than monetary mass. Its use in invoicing and the clearing of international payments are the two pointed out by Dowd and Greenaway (1993). Since a central bank does not control these two alternative measures, they are not used here as they are not directly compatible with our model. However, based on the quantitative theory of money one should expect a high degree of correlation among the three mentioned measures as more transactions require more money.

<sup>28</sup> The share of monetary aggregate of the currencies not linked to any anchor currency is negligible and is not pictured in the Figure 1.

2. Finally, the post-1979 period represents a situation in which, in the framework of our model, unquestionably  $n > 2$  and no pure-strategy equilibrium exists.

Figure 3 shows the dynamics of the key short-term interest rates at which central banks of the anchor currencies lend to commercial banks (discount rates). The figure also captures all the changes in interest rates that central banks performed during the period under research. In this respect, the plotted changes in interest rates reflect the first stage of our model. The plots allow us to detect changes in the repositioning of the central banks of anchor currencies in one-dimensional space. Short-term interest rates vary extensively, but the differences among the short-term rates are relatively small.<sup>29</sup> We observe broad medium-term parallel movements which are attributable to changes in global economic conditions. Short-term movements are frequently not parallel, which is consistent with our model's prediction for three anchor currencies with relatively close preferred policies. Short-term non-parallel policy changes are consistent with repositioning in the absence of a pure-strategy equilibrium.

In our model introduced in Section 2 we assume that the central bank of an anchor currency uses the interest rate as a major factor forming the latent (unknown) policy instrument that is used for positioning purposes within the one-dimensional space. We provide evidence on the dynamics of such repositioning in the following regressions. Recall that the most preferred locations of satellite currencies are distributed with respect to a density  $f$  on  $[0,1]$ . At each period when any central bank of the key currency changes its interest rate  $r$  we ordered the values of all three (anchor currency) interest rates as minimum, median and maximum. Then we calculated the inter-period change in the interest rate in each category between two consecutive periods  $t$  and  $t-1$  as  $\Delta \min r_t = (\min r_t - \min r_{t-1})$ ,  $\Delta \text{med} r_t = (\text{med} r_t - \text{med} r_{t-1})$ , and  $\Delta \max r_t = (\max r_t - \max r_{t-1})$ . From an econometric point of view the inter-period change in interest rate corresponds to a first difference. Finally we expressed the

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<sup>29</sup> Among the interest rate movements the recent dynamics in Japan's rate since 1995 stands out. It is a reflection of the heavy "yen carry trade"; we already described the carry trade phenomenon in footnote 9 in Section 2. The "yen carry trade" has reportedly been a fairly widespread strategy since the yen started its declining trend in the spring of 1995. Investors could borrow cheaply in the Japanese money market and invest the proceeds in a wide array of assets ranging from U.S. Treasuries to high-yielding emerging market securities. In fact, in 1996 and early 1997 high interest rates in East Asian money markets and stable dollar exchange rates attracted much of these funds. This strategy paid off as long as the yen did not appreciate and other East Asian currencies remained firmly pegged to the U.S. dollar. For more details as well as different trading strategies see Béranger et al. (1999) and Galati and Melvin (2004).

difference of each category interest rate as a function of all three categories with a one period time lag. For example, in case of the inter-period difference in minimum values the specification is  $\Delta \min r_t = \omega + \eta \min r_{t-1} + \delta \text{med}r_{t-1} + \chi \max r_{t-1} + \varepsilon_t$ . The results of the three regressions are given in Table 1.

The predictions of our model with respect to the repositioning of central banks correspond to the following regression results:

1. The inter-period change of the median values of the interest rate ( $\Delta \text{med}r_t$ ) does not depend on its own lagged value as it is also irrelevant according to our model, but it reflects the lagged minimum and maximum interest rate changes.
2. The inter-period change of the minimum values ( $\Delta \min r_t$ ) reflects the lagged minimum and median interest rate changes. The result suggests that the minimum is moving in the same direction as the changes in median values as well as changes in the minimum itself. This is consistent with the model as for the minimum values the median is the closest relevant neighboring indicator of movements within the policy space. Since the minimum is a natural lower boundary for repositioning within the policy space, we do not expect any reaction with respect to a change in the maximum value; this is also confirmed empirically.
3. The inter-period change of maximum values ( $\Delta \max r_t$ ) chiefly reflects the lagged change in the maximum value since the maximum is an upper bound of the policy space that is theoretically infinite but an interest rate of infinity is ruled out in practice. The negative sign of the coefficient associated with the lagged maximum reflects adjustments related to limiting the extent of the policy space from above. Alternatively, the movement can be interpreted as a tendency to revert to the mean: the higher a change in maximum in the last period, the more it decreases in the subsequent period. Further, the change of maximum values depends on the lagged change in minimum values that illustrate how the lower band of the policy space moves with respect to the limit of the upper bound. The two findings interpreted jointly reflect the situation in which the policy space narrows from the lower end to compensate for adjustments at its upper end. The observed changes in interest rates implicitly reflect the repositioning of central banks in the policy space in the first stage of the game. However, we admit that the observed specific links cannot be derived from our model as the model does not predict the order in which the

banks adjust their positions in the policy space. On the other hand, the findings broadly support the model in that a central bank changes its policy depending also on the other banks' policies.

The evidence given above provides indirect support of the predictions of our theoretical model concerning the behavior of satellite currency countries. Our results have provocative implications with respect to recent developments. As the euro has gained in value against the dollar, central banks in Japan, China, and other Asian countries have bought dollars to hold down the value of their own currencies. The total reserves of the four largest Asian economies—China, Japan, South Korea, and Taiwan—have more than doubled over the 2001–2003 period and reached 1.5 trillion U.S. dollars, most of it held in American government securities. China itself, until recently (mid-2005), kept its currency tightly pegged to the U.S. dollar, which greatly upset Europe since it was not allied with the dollar.<sup>30</sup> The European Union (EU) appealed to China to let its currency float and to Japan to discontinue its interventions on the yen-dollar market. The EU's rationale behind these appeals was to reduce fluctuations among the exchange rates of anchor currencies. Our conclusions would indicate just the opposite. In fact, if China kept its link to the dollar and Japan pegged the yen in some way, our model predicts that the overall situation would lean towards a stable two-currency equilibrium.<sup>31</sup> China's recent move of pegging its currency to a currency basket represented most heavily by the dollar, euro, and yen instead maintains the unstable status quo in terms of our model's predictions.<sup>32</sup>

#### **4. Concluding remarks**

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<sup>30</sup> China has kept its currency, the yuan, virtually fixed at 8.28 CNY/USD for the last twelve years. China's central bank adjusted the yuan's value to 8.11 CNY/USD on July 21, 2005 and announced that it would manage it by reference to a basket of the currencies of its main trading partners. The composition of the basket was revealed on August 10, 2005: it is dominated by the U.S. dollar, the euro, the Japanese yen and the South Korean won, and also contains the Australian, Canadian and Singapore dollars, the British pound, the Malaysian ringgit, the Russian rouble and the Thai baht. It is estimated (by Deutsche Bank) that the U.S. dollar is the largest component of the basket, with a 30 percent weight. The euro and yen likely take up 20 percent each and the won 10 percent.

<sup>31</sup> This is in line with recent arguments made by McKinnon (2004) and McKinnon and Schnabl (2004). Further, Girardin (2005) shows that during rapid-growth episodes a group of East Asian countries that had strong links with Japan in the 1980s instead developed strong links with China in the 1990s. The fact that the 1990s and early 2000s was a very special period for the Japanese economy should be kept in mind, though.

<sup>32</sup> When we extend the exposition one step further, our model offers an explanation for why Japan is not simply monetizing its debt. According to our model, if they did, one would expect them to lose pegged satellite currencies, which would decrease the extent of an informal currency area.

In this paper we build a spatial competition model in a two-stage game setup to assess whether stable coexistence, in the sense of a pure-strategy equilibrium, among the world's leading currencies is attainable. We conclude that a stable equilibrium among the existing anchor currencies is not likely to be achieved under existing monetary arrangements. We show that although a stable equilibrium can arise in the case of two anchor currencies, instability is a prominent feature in the case of three anchor currencies. While the model is a simplification of reality, in this stylized world we hint at the right number of anchor currencies. Outside this stylized world our findings might still hold, but different factors may influence the stability of the equilibrium. One issue our model does not address is why a specific number of anchor currencies might exist in the first place. A more general model would endogenize the number of anchor currencies.

We support the assumptions and implications of our model with both quantitative evidence and formal statistical inference. Our empirical results back up the predictions of our theoretical model concerning the behavior of satellite currency countries. We document large changes in the extent of the authoritative domain of anchor currencies as satellite currencies altered their ties to anchor currencies over time. Implicitly, we witness how the monetary world has changed during the past four decades.

Our results have implications for international portfolio diversification in which stable exchange rates represent lower costs incurred by an investor in terms of less hedging against unpredictable fluctuations. A reduced number of anchor currencies reduces the number of potential incidents against which an investor hedges. Thus, as hedging against exchange rate risks is important and expensive, our results suggest that moving from three to two anchors would greatly increase stability and hence reduce hedging costs. Furthermore, in the spirit of the above our results offer a possible illumination of the “home bias” hypothesis related to a lower-than-expected amount of investments made by domestic investors in international assets.

Since firms, traders, and countries currently recognize three anchor currencies and their economic behavior reflects this, we may expect further fluctuations among them as well as continuing disagreement on their overvaluation or undervaluation. Despite the stylized character of our model, an essential implication of the absence of a stable equilibrium among the policies of anchor currency countries is that exchange rates between these countries are unstable as well. Three leading currencies are probably too

many for our world, or maybe too few. The future will tell.

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Figure 1. Exchange Rate Deviations in G(3): 1964-2006. (March 1973 = 100)

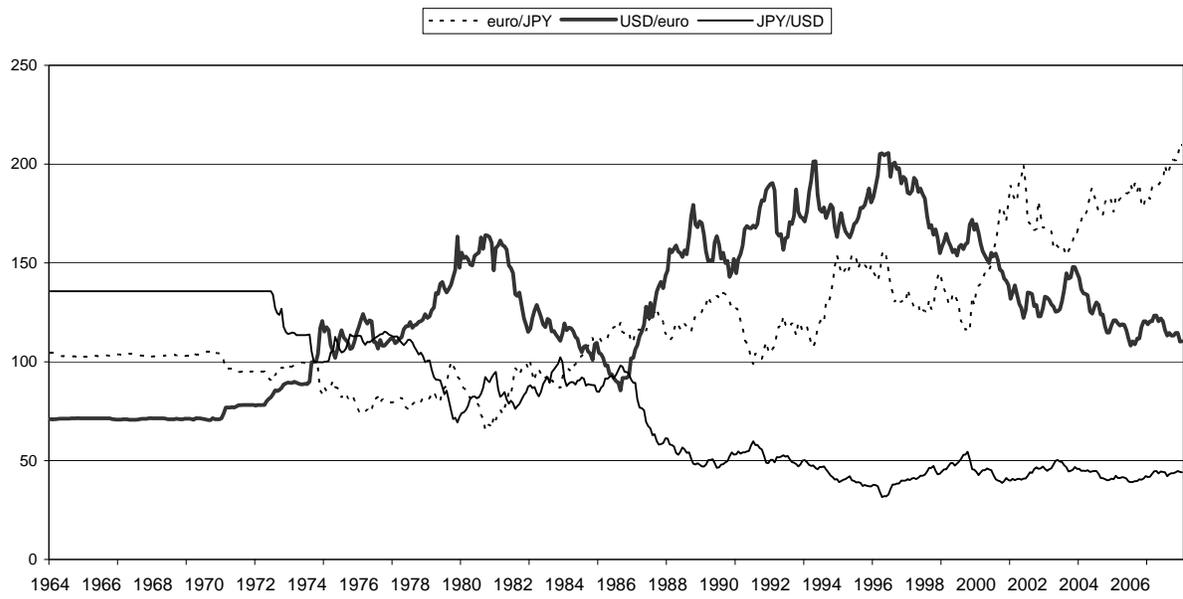


Figure 2. Relative Share of Money Aggregates Linked to Anchor Currencies (1964-2006)

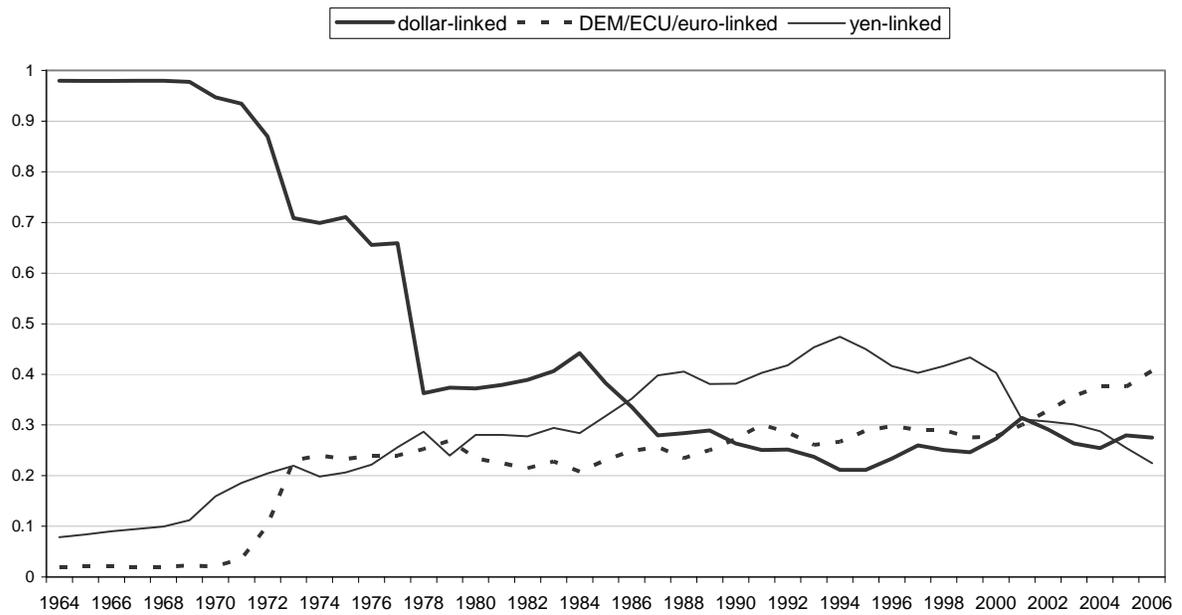


Figure 3. Discount Interest Rates of the Anchor Currencies' Central Banks (1964-2006)

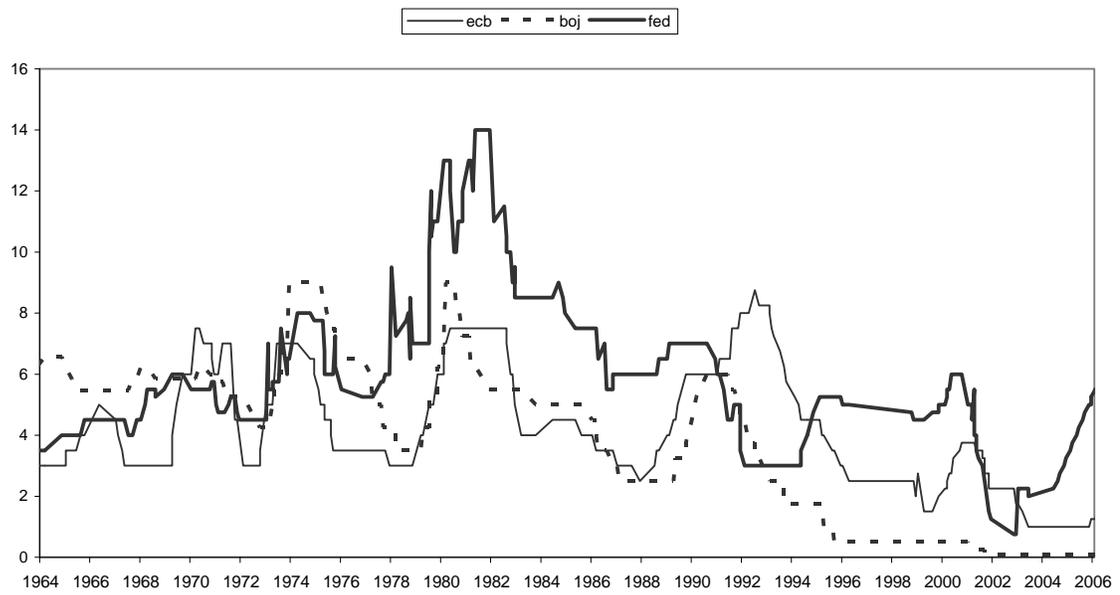


Table 1. Repositioning of Central Banks in the Policy Space: Evidence from Discount Rates Setting

Dependent variable	Explanatory variables (RHS)			Other characteristics
	$\Delta \min r_{t-1}$	$\Delta \text{med } r_{t-1}$	$\Delta \max r_{t-1}$	
$\Delta \text{med } r_t$	0.210 <sup>***</sup> (.059)	0.020 (.054)	0.103 <sup>***</sup> (.032)	$R^2=0.07, n=322$ $F=8.09^{***}$
$\Delta \max r_t$	0.202 <sup>**</sup> (.101)	0.126 (.093)	-0.192 <sup>***</sup> (.054)	$R^2=0.05, n=322$ $F=6.02^{***}$
$\Delta \min r_t$	0.118 <sup>**</sup> (.055)	0.178 <sup>***</sup> (.050)	0.016 (.029)	$R^2=0.06, n=322$ $F=6.63^{***}$

Note: \*\* and \*\*\* denote statistical significance at the 1% and 5% test levels, respectively.

APPENDIX (not for publication)

**Proof of Proposition 1.**

(a) Assume  $x_1 = x_2$ . In that case all satellite countries choose  $c_1 = c_2 = \frac{1}{2}$ . Hence  $s_1 = s_2 = \frac{1}{2}$ . By deviating to  $x_1 - \varepsilon$  or  $x_1 + \varepsilon$  with  $\varepsilon > 0$  very small, central bank 1 can capture  $s_1 = x_1 - \varepsilon$  or  $1 - x_1 - \varepsilon$  at a minimal increase in location costs. Hence unless  $x_1 = \frac{1}{2}$ , central bank 1 has an incentive to deviate (as has central bank 2). Thus the only possible equilibrium with  $x_1 = x_2$  is  $x_1 = x_2 = \frac{1}{2}$ .

This is an equilibrium if the location costs are not too high for any of the banks. Consider first the case  $p_1 < \frac{1}{2} < p_2$ . Observe that if  $x_1 < x_2$ ,  $s_1 = x_1 + \int_{x_1}^{x_2} \frac{x_2 - z}{x_2 - x_1} dz = x_1 + \frac{x_2 - x_1}{2} = \frac{x_1 + x_2}{2}$  and hence  $\frac{\partial s_1}{\partial x_1} = \frac{1}{2}$ . Thus also the derivative of  $s_i$  from the left at  $x_1 = x_2 = \frac{1}{2}$  equals  $\frac{1}{2}$ . Since  $G_i'(x_i) = s_i'(x_i) - L'(x_i - p_i)$ , we get for the derivative from the left  $G_1'(\frac{1}{2}) > 0$  if  $L'(\frac{1}{2} - p_1) = 2a(\frac{1}{2} - p_1) \leq \frac{1}{2}$  or  $p_1 \geq \frac{1}{2} - \frac{1}{4a}$ . In that case, 1 has no incentive to marginally deviate from  $x_1 = \frac{1}{2}$  to  $x_1 < \frac{1}{2}$  if  $x_2 = \frac{1}{2}$  since its loss in  $s_1$  would not be compensated by a sufficient reduction of location costs. Since  $L'' > 0$  a deviation to any  $x_1 < \frac{1}{2}$  would not pay. Clearly, a deviation to  $x_1 > \frac{1}{2}$  does not pay, because it would yield a smaller  $s_1$  at higher location costs. Likewise, we derive for  $p_2 > \frac{1}{2}$  that the necessary and sufficient condition for 2 not to deviate to  $x_2 > \frac{1}{2}$  if  $x_1 = \frac{1}{2}$  is  $L'(\frac{1}{2} - p_2) = 2a(\frac{1}{2} - p_2) \geq -\frac{1}{2}$  or  $p_2 \leq \frac{1}{2} + \frac{1}{4a}$  (since  $\frac{\partial s_2}{\partial x_2} = -\frac{1}{2}$  for  $x_2 > x_1$ ).

Similarly, if  $p_2 > p_1 > \frac{1}{2}$  then the condition for bank 1 changes to  $L'(\frac{1}{2} - p_1) \geq -\frac{1}{2}$  (which always holds if  $L'(\frac{1}{2} - p_2) \geq -\frac{1}{2}$  since  $L'' > 0$ ) and for  $\frac{1}{2} > p_2 > p_1$  the condition for bank 2 changes to  $L'(\frac{1}{2} - p_2) \leq \frac{1}{2}$  (which always holds if  $L'(\frac{1}{2} - p_1) \leq \frac{1}{2}$  since  $L'' > 0$ ).

(b) If  $p_1 < x_1 < x_2 < p_2$  and  $L'(x_1 - p_1) = \frac{1}{2}$  ( $\Leftrightarrow 2a(x_1 - p_1) = \frac{1}{2} \Leftrightarrow x_1 = p_1 + \frac{1}{4a}$ ) and  $L'(x_2 - p_2) = -\frac{1}{2}$  ( $\Leftrightarrow 2a(x_2 - p_2) = -\frac{1}{2} \Leftrightarrow x_2 = p_2 - \frac{1}{4a}$ ) then since, as was shown above,  $\frac{\partial s_1}{\partial x_1} = \frac{1}{2}$  and  $\frac{\partial s_2}{\partial x_2} = -\frac{1}{2}$ ,  $G_1'(x_1) = G_2'(x_2) = 0$  and hence neither bank 1 nor bank 2 has an incentive to marginally deviate (note that  $L'' > 0$  implies that if there is no incentive for a marginal deviation, there is also no incentive for a larger deviation that preserves  $x_1 < x_2$ ). In this case,  $x_1 < x_2$  is obviously equivalent to  $p_2 - p_1 > \frac{1}{2a}$ .

Bank 1 would want to deviate from  $x_1$  to  $x_2 + \varepsilon$  only if  $(1 - x_2) - (x_1 + \frac{x_2 - x_1}{2}) > L(x_2 - p_1) - L(x_1 - p_1)$ , that is, the additional gain in currency holdings by switching to (a position slightly to the right of)  $x_2$  will overcompensate the increase in location costs.<sup>33</sup> Bank 1 would certainly not want to deviate to any larger  $x$ , because this would imply a smaller share at higher location costs. Note that there can only be an incentive to deviate to  $x_2 + \varepsilon$  if the preferred locations of the two anchor currency central banks are relatively close together but off the median of  $f$ . In other

<sup>33</sup> In the following, we will ignore  $\varepsilon$  in the share and also in the costs because it can be arbitrarily small.

words, such an equilibrium exists, if the preferred locations of both banks are located rather symmetrically around  $\frac{1}{2}$ , are relatively far apart, or location costs are high. Note that  $L'(0) = 0$  implies  $p_1 < x_1 < x_2 < p_2$  since each bank would be willing to incur some location costs in order to increase its share  $s_i$ .

By somewhat tedious, but straightforward computation we can show that  $(1 - x_2) - (x_1 + \frac{x_2 - x_1}{2}) \leq L(x_2 - p_1) - L(x_1 - p_1) \Leftrightarrow 1 - p_2 - p_1 \leq a(p_2 - p_1)^2 - \frac{1}{4a}$  (note that the right-hand side is  $> 0$  since  $p_2 - p_1 > \frac{1}{2a}$ ).

Similarly, bank 2 has no incentive to deviate to  $x_1 - \varepsilon$  if  $x_1 - (1 - x_2 + \frac{x_2 - x_1}{2}) \leq L(x_1 - p_2) - L(x_2 - p_2)$  which is equivalent to  $p_2 + p_1 - 1 \leq a(p_2 - p_1)^2 - \frac{1}{4a}$ . Hence no bank has an incentive to deviate if  $|1 - p_1 - p_2| \leq a(p_2 - p_1)^2 - \frac{1}{4a}$ .

(c) As was shown in (a) no equilibrium exists with  $x_1 = x_2 \neq \frac{1}{2}$ . Point (b) states necessary and sufficient conditions for an equilibrium with  $p_1 < x_1 < x_2 < p_2$ . It is obvious that bank 1 would profit from deviating from an  $x_1$  with  $x_1 < p_1 < x_2$ ,  $x_1 < x_2 < p_1$ ,  $x_2 < p_1 < x_1$  or  $p_1 < x_2 < x_1$  because 1 could simultaneously increase  $s_1$  and lower location costs. Similarly,  $x_2 < p_2 < x_1$ ,  $p_2 < x_1 < x_2$ ,  $x_1 < p_2 < x_2$  and  $x_2 < x_1 < p_2$  are impossible. This covers all possible constellations of locations. If there is a “smallest policy unit  $\varepsilon$ ”, then there could in principle be constellations  $x_1 = x_2 - \varepsilon < p_1$ . Bank 1 would then not wish to deviate to  $x_2$  (or anything larger) if  $s_1 > s_2$ . In that case, however, bank 2 would want to deviate to  $x_1$  (as long as  $\varepsilon$  is small enough such that the increase in location costs is negligible). This situation leads to the requirement  $\frac{1}{2} - \varepsilon \leq x_1 \leq \frac{1}{2}$  and we are essentially back in case (a). The remaining cases with the smallest possible policy unit would be solved in a similar way. QED

## Proof of Proposition 2

(a) **Step 1:** bank 2 does not want to deviate:

Note that for all  $x_2$  with  $x_1 < x_2 < x_3$ ,  $s_2 = \frac{1}{2}(x_3 - x_1)$  and, hence, bank 2 has no incentive to deviate to any such  $x_2$  since it will not affect  $s_2$  but will cause positive location costs. If bank 2 deviates to  $x_2 = x_1 - \varepsilon$ , then its share is  $x_1$  (we will again ignore  $\varepsilon$  in the share and also in the costs because it can be arbitrarily small). Deviating does not pay, therefore, if  $x_1 - s_2^* \leq L(x_1 - p_2)$  which is (as again tedious but straightforward computation shows) equivalent to the first part of (3). If bank 2 deviates to  $x_1$ , then  $s_2 = \frac{s_2^*}{2} + \frac{x_1}{2}$ , so if deviating to  $x_1 - \varepsilon$  does not pay, deviating to  $x_1$  definitely does not pay. If bank 2 deviates to  $x_2 = x_3 + \varepsilon$  then  $s_2 = 1 - x_3$ ; so deviating does not pay if  $1 - x_3 - s_2^* \leq L(x_3 - p_2)$ , which is equivalent to the first part of (4). If bank 2 deviates to  $x_3$ , then  $s_2 = \frac{s_2^*}{2} + \frac{1 - x_3}{2}$ ; so if deviating to  $x_3 + \varepsilon$  does not pay, then deviating to  $x_3$  definitely does not pay.

**Step 2:** bank 1 does not want to deviate:

Since  $L'(x_1 - p_1) = L'(\frac{1}{4a}) = \frac{1}{2}$  the marginal location costs of bank 1 at  $x_1$  are equal to

the marginal gain in  $s_1$ , hence bank 1 has no incentive to marginally deviate and condition (1) is equivalent to  $x_1 \leq p_2$ . Since  $L'' > 0$ , bank 1 has no incentive to deviate to any  $x < p_2$ .

Bank 1 does not want to deviate to any  $x$  with  $p_2 < x < x_3$ : Note that  $s_1 = \frac{x_3 - p_2}{2}$  for all such  $x$ . Hence bank 1 would, if anything choose  $p_2 + \varepsilon$ . Bank 1 will not deviate to  $p_2 + \varepsilon$  if  $\frac{x_3 - p_2}{2} - \frac{p_2 + x_1}{2} \leq L(p_2 - p_1) - L(x_1 - p_1)$ , which is equivalent to the second part of (3).

Bank 1 does not want to deviate to  $x_3 + \varepsilon$  because bank 2 does not want to deviate to  $x_3 + \varepsilon$ , as can be seen by the following argument. For ease of notation let  $A = x_1$ ,  $B = p_2 - x_1$ ,  $C = x_3 - p_2$  and  $D = 1 - x_3$ . Assume that bank 1 wants to deviate to  $x_3 + \varepsilon$ , i.e.  $D - A - \frac{B}{2} > a(x_3 - p_1)^2 - a(x_1 - p_1)^2$ , but bank 2 does not, i.e.  $D - \frac{C+B}{2} \leq a(x_3 - p_2)^2$ . Observe that

$$\begin{aligned} a(x_3 - p_1)^2 - a(x_1 - p_1)^2 &= a(x_3 - p_2 + p_2 - p_1)^2 - a(x_1 - p_1)^2 = \\ &= a(x_3 - p_2)^2 + a(p_2 - p_1)^2 - a(x_1 - p_1)^2 + 2a(x_3 - p_2)(p_2 - p_1) \geq a(x_3 - p_2)^2 + \frac{C}{2} \end{aligned}$$

Hence, the above assumptions imply that  $D - A - \frac{B}{2} > a(x_3 - p_2)^2 + \frac{C}{2} \geq D - \frac{B}{2}$ , which can obviously not be true.

By deviating to  $p_2$ , bank 1 would obtain the average of the shares that it obtains at  $p_2 - \varepsilon$  and  $p_2 + \varepsilon$ , so if it does not want to deviate to either of these, it does not want to deviate to  $p_2$  either, and by a parallel argument it does not want to deviate to  $x_3$ .

Deviating to any other location is dominated because it yields the same or a lower  $s_1$  at a higher location cost than one of the options discussed above.

**Step 3:** bank 3 does not want to deviate:

The situation of bank 3 is symmetric to that of bank 1 and hence the conditions are derived in a completely parallel way.

The above analysis shows that conditions (1) to (4) are sufficient for  $(x_1, p_2, x_3)$  being an equilibrium, but also necessary for an equilibrium with  $x_1 < p_2 < x_3$  and  $x_1 < x_2 < x_3$ .

**(b) Step 1:** banks 1 and 2 do not want to deviate: Note that since  $3x_1 = x_3$  we have  $x_1 = \frac{x_3 - x_1}{2}$ , so deviations to  $x$  with  $x_1 < x < x_3$  also yield  $s = \frac{x_3 - x_1}{2} = x_1$  but since  $p_1 < p_2 \leq x_1$  the location costs are higher. A deviation to  $x < x_1$  implies a reduction of  $s$  by  $\frac{x_1 - x}{2}$ . Since  $x_1 - p_2 < x_1 - p_1 \leq \frac{1}{4a}$ , we have  $L'(x_1 - p_2) < L'(x_1 - p_1) \leq \frac{1}{2}$ . Thus the decrease in location costs is smaller than the loss in  $s$  and a deviation to  $x < x_1$  does not pay. Finally, a deviation to  $x_3 + \varepsilon$  does not pay for bank 2 if  $1 - x_3 - x_1 \leq L(x_3 - p_2) - L(x_1 - p_2)$ , which is equivalent to (5). Since  $L'' > 0$  and  $p_1 < p_2$ , deviating to  $x_3 + \varepsilon$  does not pay for bank 1 if it does not pay for bank 2.

**Step 2:** bank 3 does not want to deviate: since  $x_3 = p_3 - \frac{1}{4a}$ ,  $L'(x_3 - p_3) = -\frac{1}{2}$  and hence bank 3 does not want to deviate to any  $x > x_1$ . Since  $s_3^* > \frac{1}{3} > x_1$ , a deviation to  $x_1 - \varepsilon$  or

$x_1$  implies a lower share at a higher location cost and hence bank 3 has no incentive to deviate.

(c) This is just the symmetric situation to (b). The proof is essentially identical.

(d) There are no further equilibria.

**Step 1:** as established above, the equilibrium in (a) is the only equilibrium with  $x_1 < p_2 < x_3$  and  $x_1 < x_2 < x_3$ . There can be no equilibrium with  $x_1 < x_2 < x_3$  but  $p_2 \leq x_1$ , because in that case bank 2 could, by deviating to  $x$  with  $x_1 < x < x_2$ , obtain the same  $s_2$  at lower location costs.<sup>34</sup> By a parallel argument, there is also no equilibrium with  $x_1 < x_2 < x_3$  but  $x_3 \leq p_2$ . Hence the equilibrium in (a) is the only equilibrium with  $x_1 < x_2 < x_3$ .

**Step 2:**  $x_1 = x_2 < x_3$  implies  $x_1 = \frac{x_3 - x_1}{2}$  otherwise bank 1 or 2 could, by a marginal deviation, increase its share at essentially 0 increase in location costs. This then implies  $p_1 < p_2 \leq x_1$  because any  $x$  with  $x_1 < x < x_3$  yields the same share, so if  $p_2 > x_1$ , bank 2 could obtain the same share at lower location costs. Hence the equilibrium in (b) is the only equilibrium with  $x_1 = x_2 < x_3$ .

**Step 3:** by the same argument as in step 2, the only equilibrium with  $x_1 < x_2 = x_3$  is the equilibrium in (c).

**Step 4:**  $x_1 = x_2 = x_3$  cannot be an equilibrium: in this case  $s_i = \frac{1}{3}$  and by a marginal deviation bank  $i$  could obtain  $\max(x_i, 1 - x_i) \geq \frac{1}{2}$ .

**Step 5:** in equilibrium  $x_2 < x_1 \leq x_3$  is impossible, because in that case  $p_1 < x_1$  or  $x_2 < p_2$  and, hence, one bank could lower its location costs while increasing or retaining its share (note that as was argued in the proof of part (b), if in equilibrium  $x_1 = x_3$  then  $\frac{x_3 - x_2}{2} = 1 - x_3$ , so by deviating to  $x$  with  $x_2 < x < x_3$ , bank 1 would obtain the same  $s_1$ , as it is also the case for  $x_1 < x_3$ ). On the other hand,  $x_2 = x_1 < x_3$  corresponds to the equilibrium in (b), so all cases  $x_2 \leq x_1 \leq x_3$  are covered (in case of equality of all  $x$ , step 4 applies).

**Step 6:** the argument why any constellation,  $x_1 \leq x_3 < x_2$ ,  $x_2 \leq x_3 \leq x_1$ ,  $x_3 \leq x_2 \leq x_1$ ,  $x_3 \leq x_1 \leq x_2$  cannot occur in equilibrium is the same as in step 5: at least one bank can reduce its location costs without reducing its share if at least one inequality is strict; otherwise the argument of step 4 applies.

This covers all possible constellations of  $x_1$ ,  $x_2$ , and  $x_3$  and shows that no equilibrium except for those in (a), (b), and (c) exist. QED

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<sup>34</sup> If there is a “smallest policy unit  $\varepsilon$ ” then there could be an equilibrium where  $x_1 < x_2 < x_3$  but

$p_2 \leq x_1$ , namely if  $x_2 = x_1 + \varepsilon$  and  $s_2$  is larger than  $s_1$ , but in that case bank 1 would have an incentive to deviate to  $x_2$ , unless the difference in shares is very small, so this essentially corresponds to the equilibrium in (b).